Generating Piecewise-Regular Code from Irregular Structures

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WHERE DISCOVERIES BEGIN

UNIVERSIDADE DA CORUÑA

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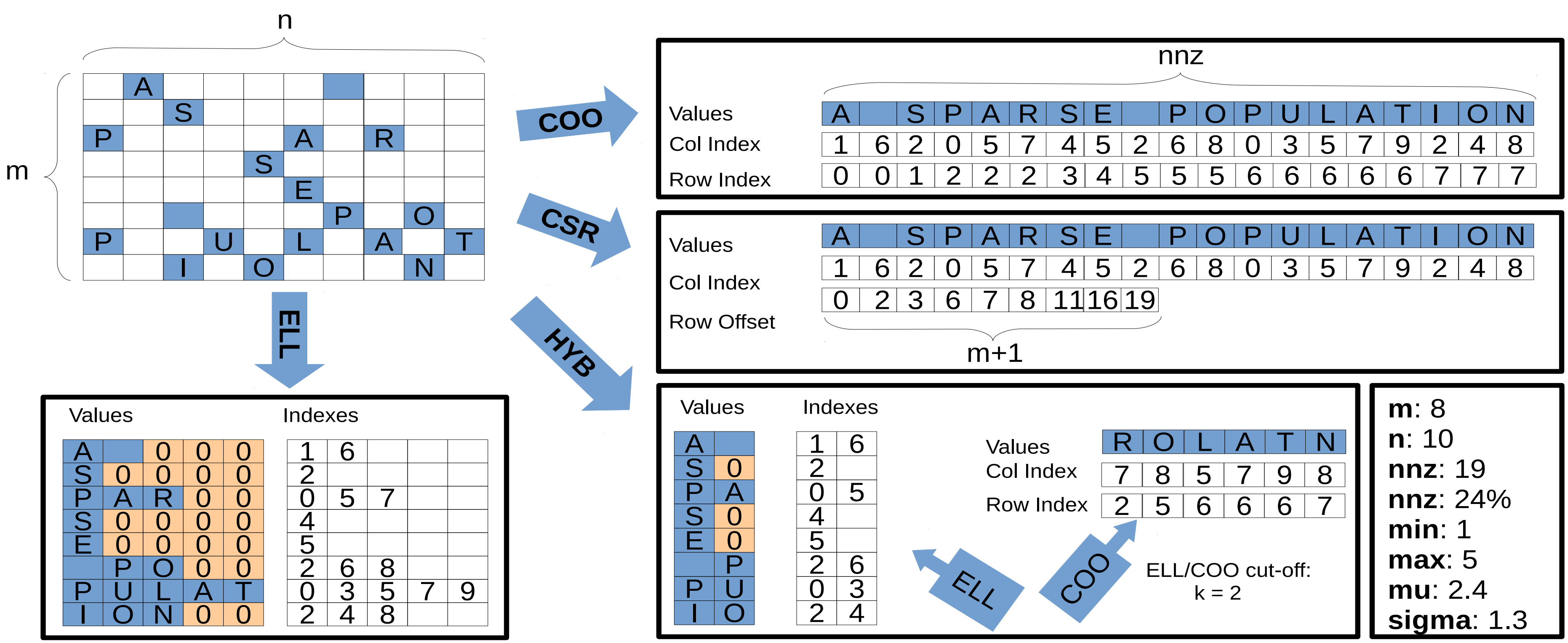
Overview

Data-specific compilation

Main idea: synthesize code that is specialized to a specific sparse data structure, using polyhedra

- **Irregular and sparse data structures are central in scientific computing and in machine learning**
  - **Graph processing, neural net inference after weight pruning, etc.**
- **Typical approach: encode the sparse structure in some format, and provide a generic executor code to traverse the data**
- **Proposed approach: encode the sparse structure with polyhedra, and generate a specialized executor code for that structure**
- **Tunable: target SIMD / performance, target compression / code size, etc.**
- **General: works for n-dimensional sparse data structures (e.g., sparse tensors)**

Sparse Data Representations



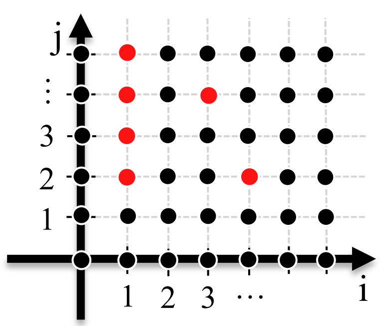
Computing on Sparse Structures

Compressed Sparse Row (CSR) code for sparse matrix vector multiply

```
for (i = 0; i < nrows; i++)
    for (j = pos[i]; j <= pos[i+1]; j++)
        y[i] += csr_data[j] * x[cols[j]];
```

- Code is generic for any sparse matrix
- For every nonzero of the matrix, performs **4** memory reads
- SIMD vectorization requires gather/scatter, code is not regular/polyhedral

Code specialized for one specific sparsity structure:



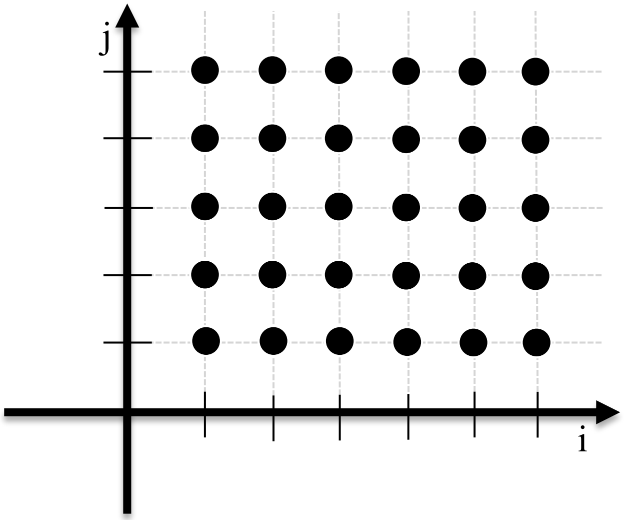

```
for (j = 2; j <= 5; j++)
   y[1] += csr_data[j-2] * x[j];
y[3] += csr_data[5] * x[4];
y[4] += csr_data[6] * x[2];
```

Application Context, Pros and Cons

- **Generating *specialized* code for one sparsity structure:**
  - Avoids the need for genericity: can remove indirection arrays / irregularity
  - Makes the loop nests easier to vectorize
  - Robust to any data changes, only the sparsity itself should not change
  - May reduce footprint, but can lead to very large code size too
  - Loses genericity: each sparse structure has a different executor program
- **Some important use cases:**
  - Sparse Matrix Vector Multiply (especially iterative SpMV)
  - Inference of some classes neural networks (especially after weight pruning)
  - Sparse tensors

But What is a Polyhedron?

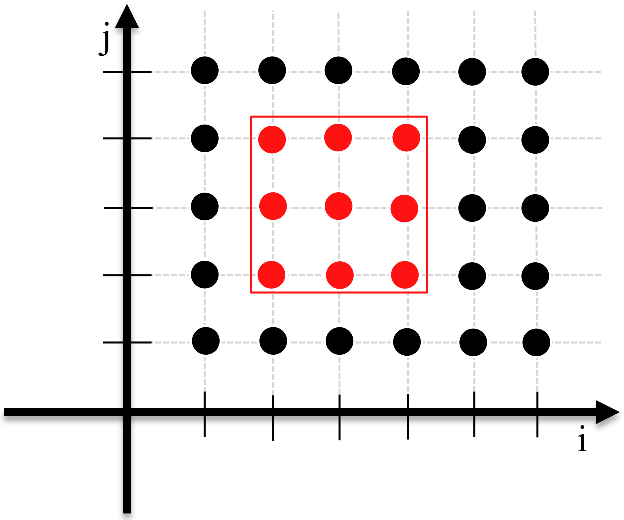
Example



Grid of 2D Integer points

But What is a Polyhedron?

Example



2D Integer points

List of points

i	j
2	2
2	3
2	4
3	2
3	3
3	4
4	2
4	3
4	4

Compact description

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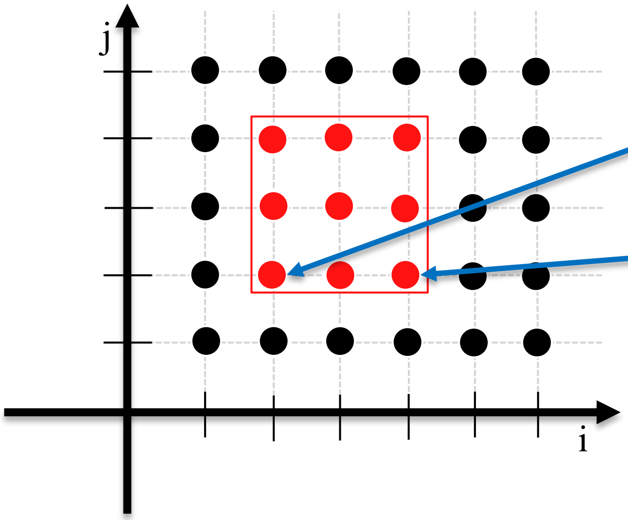

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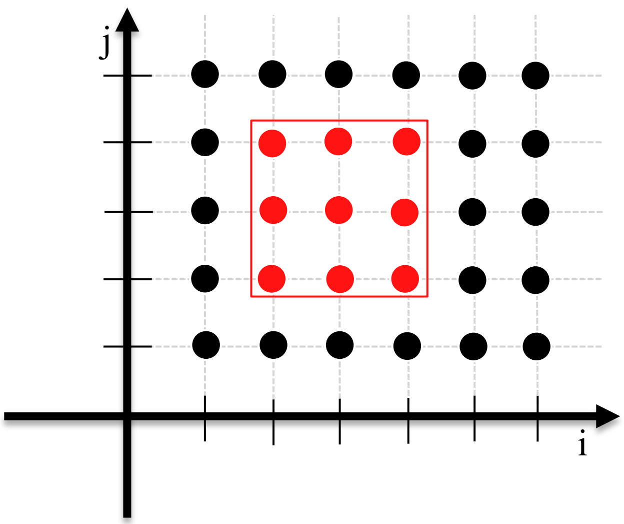

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Compact description

$$D : \{ [i,j] : 2 \leq i \leq 4 \text{ and } 2 \leq j \leq 4 \}$$

Polyhedron: described as the intersection of half-planes (e.g., $i \leq 2$), all points in the intersection are in the polyhedron

Dimensionality: 2

In this work: model only polyhedra of integer points

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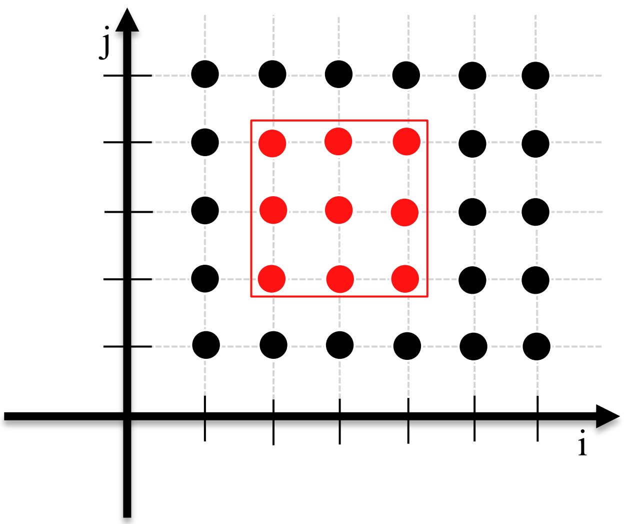

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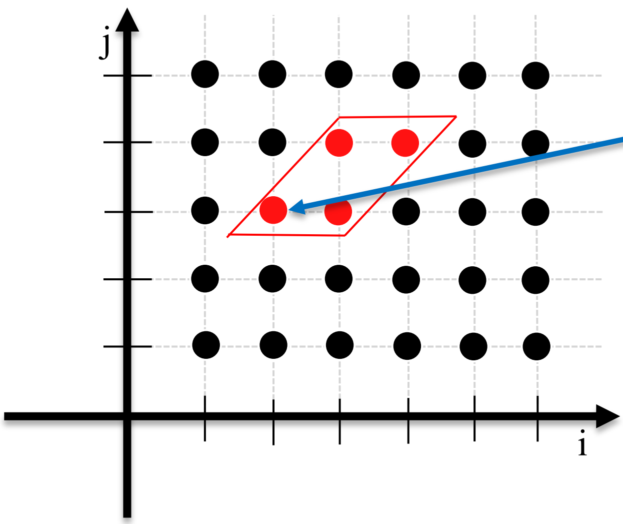
More complex shapes?

But What is a Polyhedron?

Example



2D Integer points

List of points

i	j
2	3
3	3
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Compact description

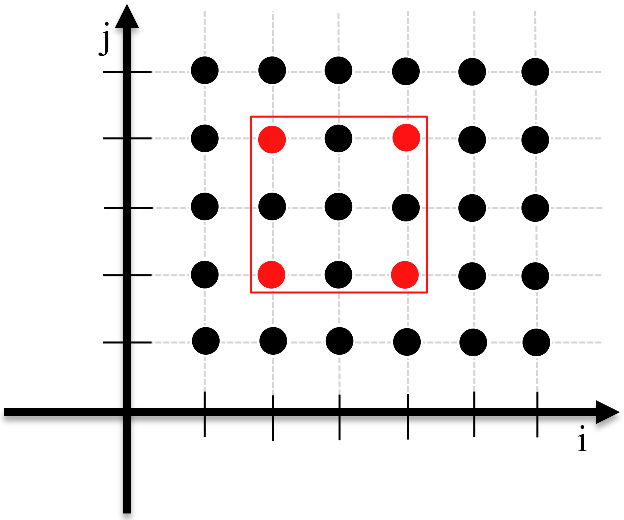
$$D : \{ [i,j] : 2 \leq i \leq 4 \text{ and } 3 \leq j \leq 4 \text{ and } j \geq i \text{ and } j \leq i+1 \}$$

Polyhedron: possibly many half planes to describe it => **affine inequalities**

Inequalities may involve several variables / dimensions

But What is a Polyhedron?

Example



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Compact description

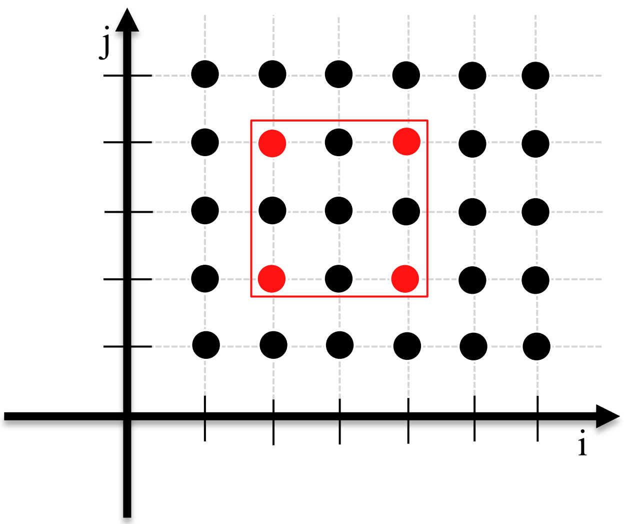
~~D : { [i,j] : 2 ≤ i ≤ 4 and 2 ≤ j ≤ 4 }~~

Still describes 9 points!!

But what about holes in the shape?

But What is a Polyhedron?

Example



2D Integer points

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i	j
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Compact description

D : { [i,j] : 1 ≤ i ≤ 2 and
1 ≤ j ≤ 2 }

+

Intersected with an integer lattice:

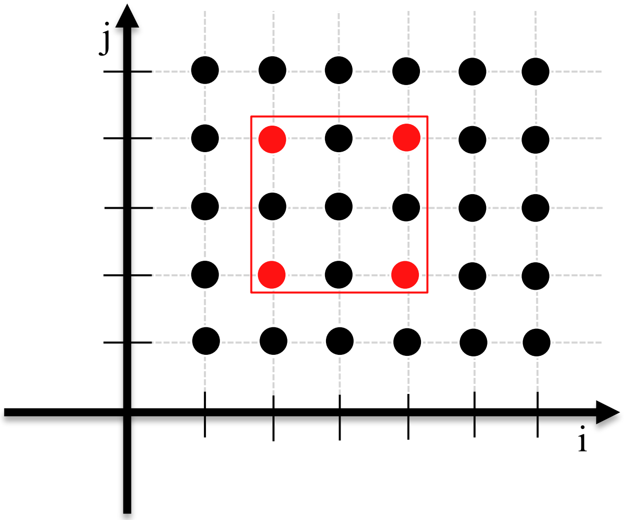
L : { [i,j] —> [x,y] : x = 2i and y = 2j }

D contains 4 points, the lattice L captures their exact coordinates (stride of 2 here)

A polyhedron intersected with a lattice is a Z-Polyhedron

But What is a Polyhedron?

Example



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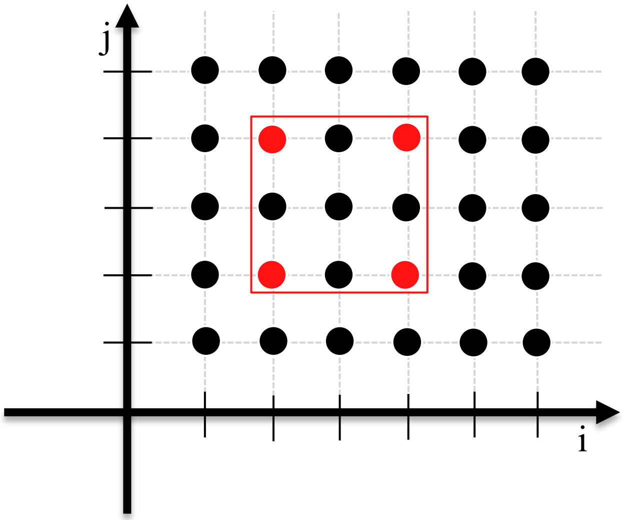
Z-Polyhedra can have "holes", needed for sparse structures

Z-Polyhedra are Code, Too

Example



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```
for (i = 1; i <= 2; i++)
  for (j = 1; j <= 2; j++)
    S(2i,2j); // x = 2i, y = 2j
```

This code traverses all and only points in the Z-polyhedron

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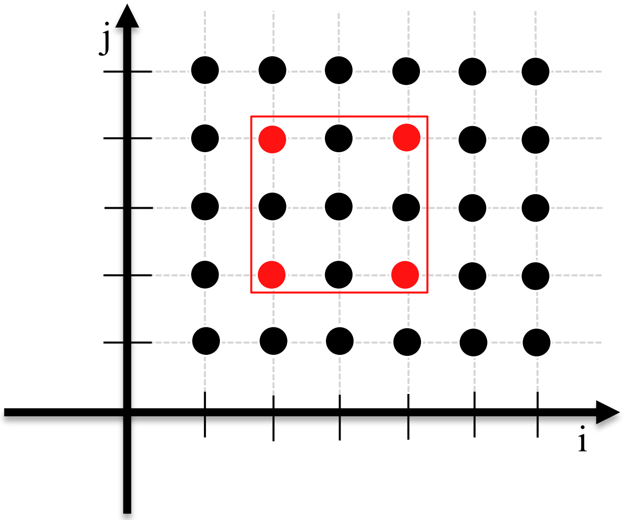

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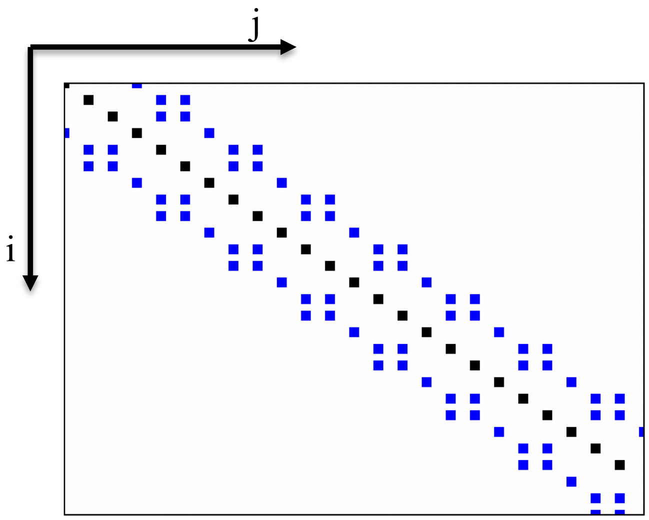
And What is a Sparse Structure?

Here, a sparse structure is simply a series of integer tuples

Example: a sparse matrix is represented by the tuple (i,j,data)



HB/nos1 matrix from SuiteSparse

	i	cols[j]	&(A_data[j])
1:	0	0	0x00
2:	0	3	0x04
3:	1	1	0x08
4:	1	4	0x0C
5:	1	5	0x10
6:	2	2	0x14
7:	2	4	0x18
8:	2	5	0x1C
9:	3	0	0x20
10:	3	3	0x24
11:	3	6	0x28
…	…	…	…

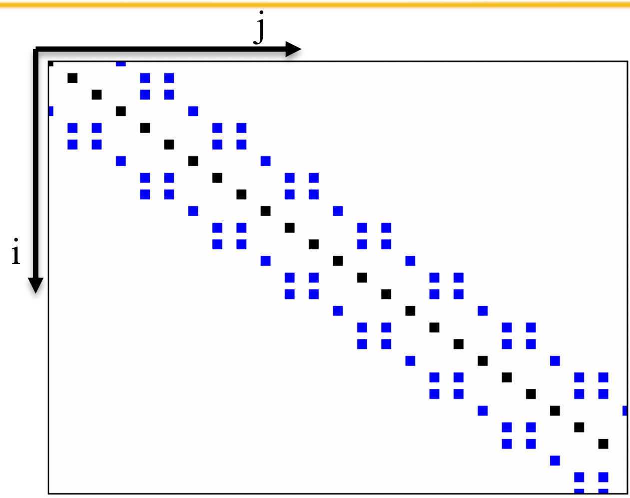
We handle sparse structures of arbitrary dimensionality, this includes sparse tensors

Representing Integer Tuples as Z-Polyhedra

- **A Z-Polyhedron models sets of integer tuples, with "holes"**
- **A sparse structure is a list of integer tuples, or points**
- **So we can represent a sparse structure as a union of Z-polyhedra!**
  - **Target scenario: many points can be captured in a single polyhedron**
  - **Performance objective: polyhedra should be easy to SIMD vectorize**

- **Challenges:**
  1. **How to determine the shapes (polyhedron and lattice) that captures the largest number of points, *efficiently*?**
  2. **How to reach good performance for e.g. SpMV programs encoded as polyhedra?**

Encoding Sparsity with Polyhedra



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8:	2	5	0x1C
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11:	3	6	0x28

D1 : { [i,j,k] : i = 2 and 4 <= j <= 5 and k = 4j + 8 }

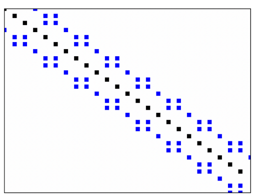
D2: { [i,j,k] : 2 <= i <= 3 and i = j and k = 16i – 12 }

When modeling problems like SpMV, we consider the trace reorderable

<u>That is, non-consecutive points in the original trace may be grouped together</u>

Complexity Trade-Offs [1/2]

- **A Z-Polyhedron may use more dimensions than the tuple size**
  - **Think tiling a 2D iteration space: you obtain a new 4D iteration space, but that still describes exactly the same original set of 2D points**

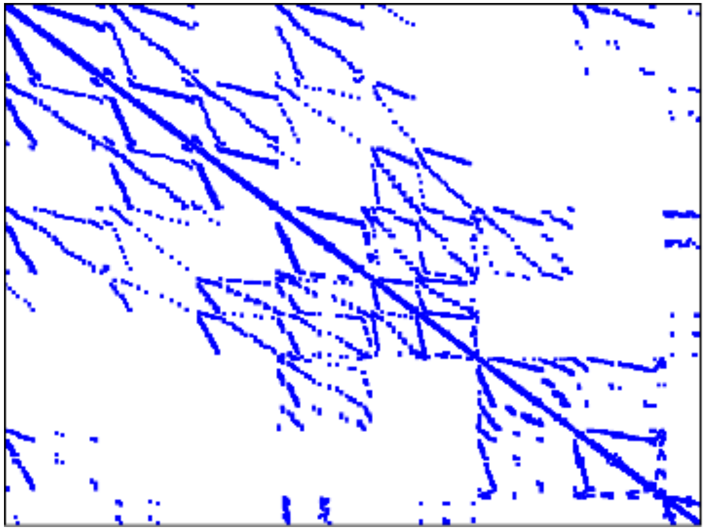


max_d	2	3	4	5	6	7	8
pieces	312	159	81	4	3	2	1
cycles	11373	11583	9938	35730	34116	39306	50371
LoC	772	1004	671	195	368	165	101

- **Using more variables/dimensions in the polyhedron (maxd) reduces the number of polyhedra needed (pieces) to capture the full matrix**
  - **Leads to better compaction (LoC)**
- **<u>But it does not necessarily lead to better performance</u>**

Complexity Trade-Offs [2/2]

- **Complex sparse structures need many polyhedra to capture them**
  - This sparse matrix, HB/can_1072 is reconstructed with 870 polyhedra, of up to 8 dimensions
  - Code size is directly related to the number of polyhedra needed

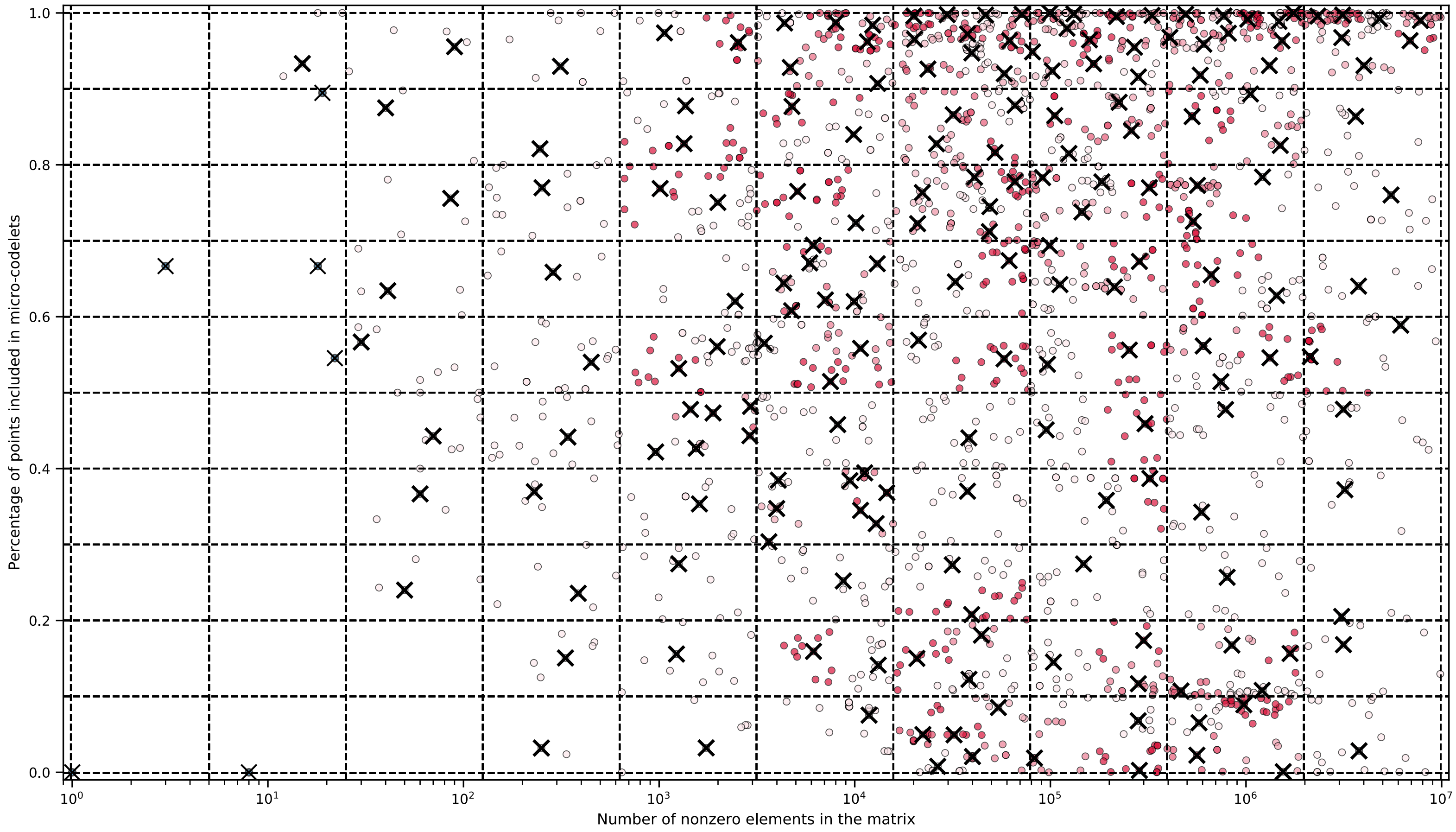


- **In this work, we design a series of algorithms that trade-off the number of polyhedra needed versus their "complexity"**
  - Try simple shape first: "rectangles", with regular strides (SIMD-friendly)
  - Try more complex shapes afterwards (skewed ones, with many dimensions)

High-Level Procedure

- 1: obtain a series of integer tuples describing the sparse structure coordinates
  - Simply scan the structure, printing the coordinates
- 2: Find simple, “rectangular” shapes by mining the trace
  - Single-level codelets: prototype shapes are chosen to be SIMD friendly
  - Implementation: mostly brute-force, but in practice extremely quick (seconds)
- 3: Try to build shapes-of-shapes, by hierarchical reconstruction
  - Create a new set of points with the polyhedra origins from 2:, and repeat!
  - Increase the complexity of shapes: use the Extended TRE algorithm for the second-level of reconstruction, as SIMD considerations are less useful here
- 4: Generate efficient code by carefully inserting code prefetch instructions
  - Code size vastly increases and exceed L1 cache, and loops often iterate over only few iterations
  - Need to explicitly prefetch the code to be executed in advance to gain performance
  - Codegen from polyhedra description is straightforwad for codelets

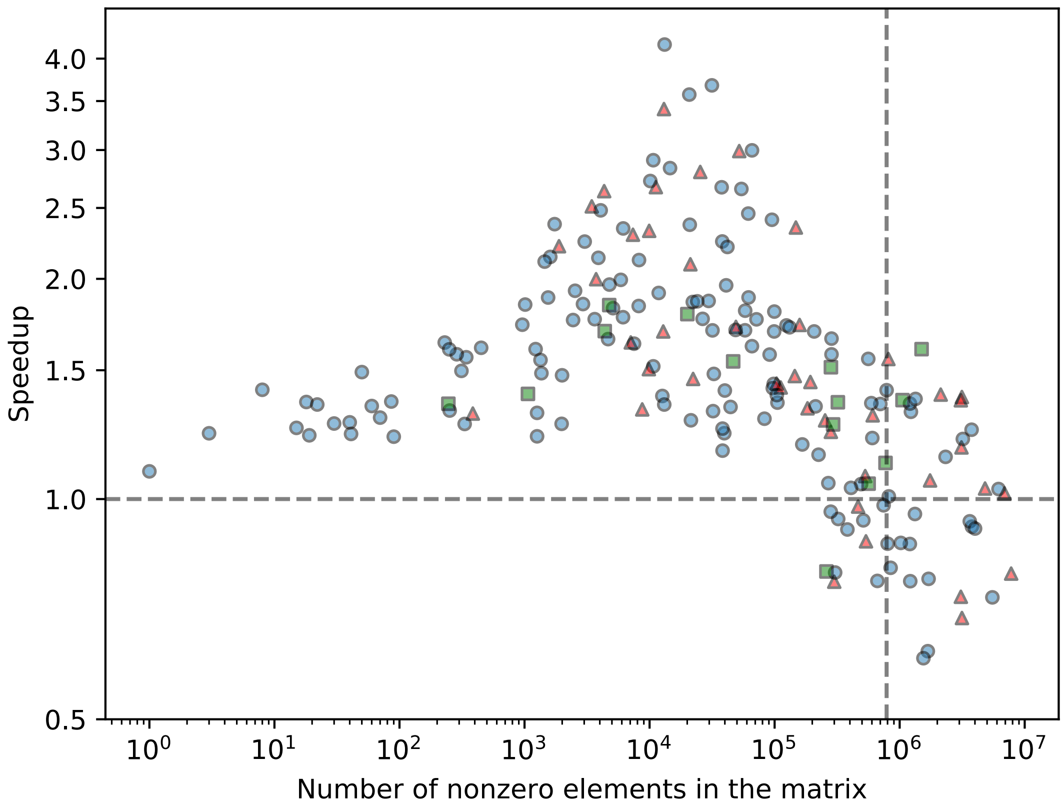
Experimental Results [1/4]



2600+ matrices from SuiteSparse with less than 10M nonzeros

We evaluate on 200 representative matrices

Experimental Results [2/4]



Experimental setup:

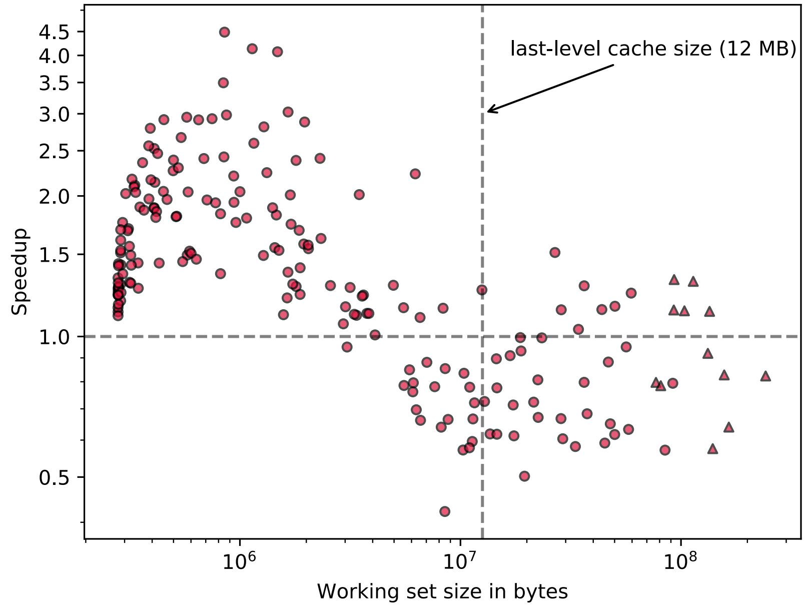
Core i7 8700k (3.7GHz)
Using hugepages
Compiled with ICC 18.03

Baselines: best of
- Vanilla SpMV C code
- Intel MKL IE

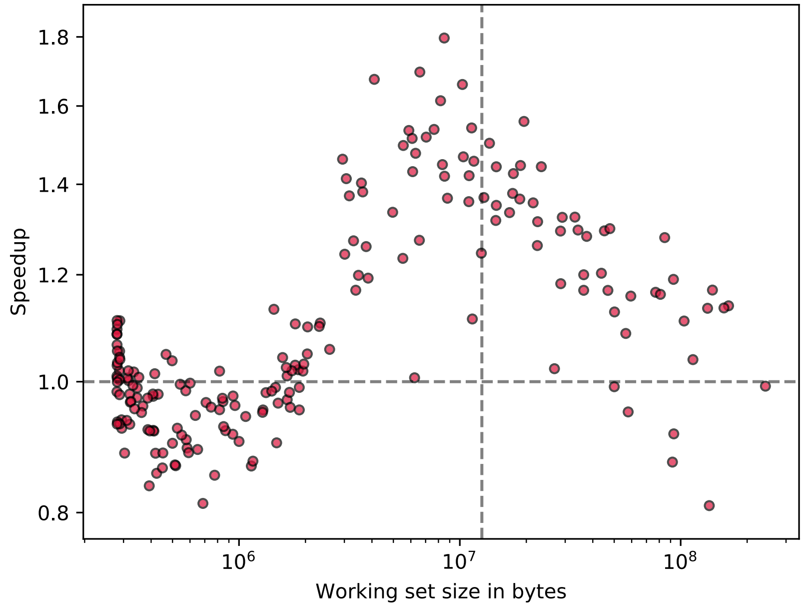
circle: single-level reconst.
triangle, square: hierarchical

- **Performance increases in the majority of cases, but not all**
  - **Complex interplay between instruction count increase, memory traffic pattern modifications, and SIMD vectorization**

Experimental Results [3/4]



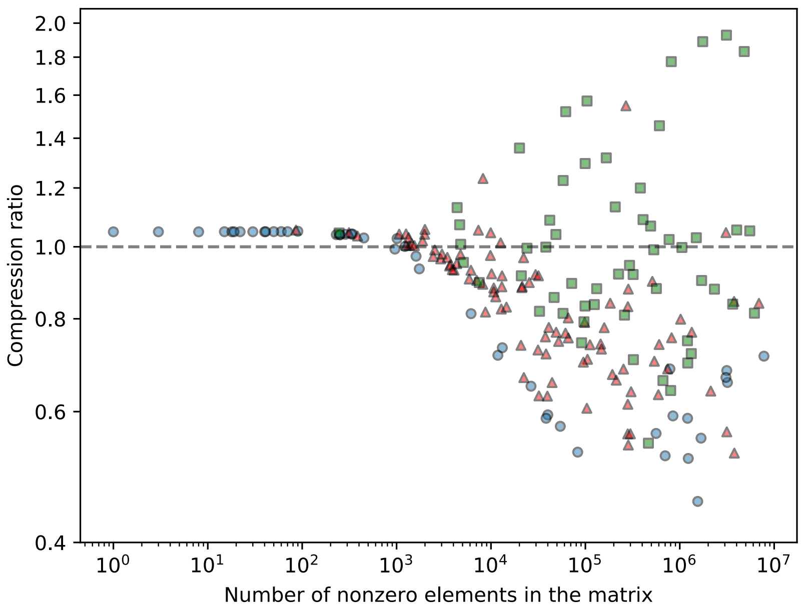
Performance **without** instruction prefetch insertion



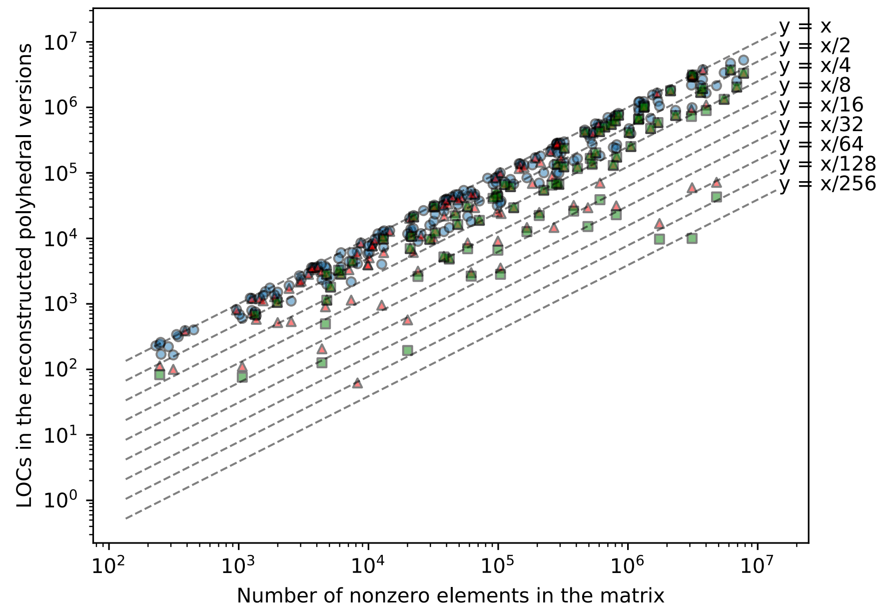
Improvement from instruction prefetch insertion alone

- **Code prefetching is critical for performance esp. for large matrices**
  - **Prefetch inserted every 64B of instructions, inserted 4kB before code is used**

Experimental Results [4/4]



Best compression achieved
(not necessarily best performance)



Generated code size versus
number of nonzeros

- **Compression ratio: CSR footprint / size of data+code generated**
  - **Best compression is achieved with different codelets, different objectives/trade-offs than for performance**

Take-Home Message

Sparse data structures using integer coordinates can be represented as a union of Z-polyhedra

- Performance improved, removal of indirection arrays, better SIMD
- May achieve compaction over other sparse formats, e.g. CSR

- **Quick synthesis time, but generated code can be very large**
- **General approach: works for sparse tensors**
  - Extensive study of 200 sparse matrices from SuiteSparse
  - Early results with neural network weight pruning (see paper)
- **Active line of work:**
  - Design of NN weight pruning aware of polyhedra shape objectives
  - Design new shape/polyhedron templates for better performance and compaction